

# Graviton Propagators, Brane Bending and Bending of Light in Theories with Quasi-Localized Gravity

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**ABSTRACT:** We derive the graviton propagator on the brane for theories with quasi-localized gravity. In these models the ordinary 4D graviton is replaced by a resonance in the spectrum of massive Kaluza-Klein modes, which can decay into the extra dimension. We find that the effects of the extra polarization in the massive graviton propagator is exactly cancelled by the bending of the brane due to the matter sources, up to small corrections proportional to the width of the resonance. Thus at intermediate scales the classic predictions of Einstein's gravity are reproduced in these models to arbitrary precision.

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Following the work [1, 2] of Randall and Sundrum (RS) there has been considerable interest [3–20] in the phenomenon of localization of gravity (for previous relevant work see [21, 22]). RS found a solution to the five dimensional Einstein equations in a background created by a single positive tension 3-brane and a negative bulk cosmological constant which reproduces the effects of four-dimensional gravity on the brane without the need to compactify the fifth dimension. The reason for this is that the fluctuations of the 4D metric are described by an ordinary quantum mechanical Schrödinger equation, where the shape of the QM potential resembles a volcano. This potential supports exactly one bound state with zero energy, which can be identified with the 4D massless graviton, since the wave functions of the massive continuum Kaluza-Klein (KK) modes are suppressed at the brane due to the tunneling through the potential barrier. Thus the effects of the KK modes are negligible for small energies, and at large distances ordinary 4D gravity is reproduced.

Recently Gregory, Rubakov and Sibiryakov (GRS) found a brane model in which 4D gravity is reproduced only at intermediate scales, since at very small scales there are the same power-law corrections as in the RS model, while at very large scales the gravitational theory on the brane is again modified [23].<sup>2</sup> It has been explained, that the reason behind this phenomenon (dubbed “quasi-localization of gravity”) is that the zero-mode of the QM system of these theories becomes unstable and the exact bound state is replaced by a resonance of zero mass in the continuum spectrum [23, 25, 26] (see also [27]). As long as the lifetime of this resonance is large, there is a large region of intermediate scales where an effective 4D Newton potential is reproduced. Once we get to large enough distances, the resonance will decay and therefore the gravitational potential will be corrected.

It has been however suggested in [26], that even though the correct Newton potential is reproduced at intermediate scales, one does not reproduce the results of ordinary 4D Einstein gravity. The argument given in [26] for this is that in these models the resonant mode is a collective effect of very light (but  $m \neq 0$ ) KK modes. It is, however, well-known that the  $m \rightarrow 0$  limit of a massive graviton propagator does not reproduce the massless graviton propagator, due to the fact that the number of polarizations of the two fields do not match [28]. Based on this discontinuity in the graviton propagator as  $m \rightarrow 0$ , it was argued in [26] that the predictions of theories with quasi-localization of gravity would significantly differ from those of ordinary gravity. In particular the bending of light was predicted to be  $\frac{3}{4}$  of the value in general relativity.

In this letter we show that this is in fact not the case. We find that up to additional small corrections (which vanish in the limit in which the width of the resonance  $\Delta m \rightarrow 0$ ) the results of ordinary 4D gravity *are* reproduced on the brane at intermediate scales as in the RS scenario.

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<sup>2</sup>A similar proposal has been made in [24].

The issue is that the 5D massless graviton propagator is given by

$$G_5(x, x')_{\mu\nu\rho\sigma} = \int \frac{d^5 p}{(2\pi)^5} \frac{e^{ip \cdot (x-x')}}{p^2} \left( \frac{1}{2} g_{\mu\rho} g_{\nu\sigma} + \frac{1}{2} g_{\mu\sigma} g_{\nu\rho} - \frac{1}{3} g_{\mu\nu} g_{\rho\sigma} \right) , \quad (1)$$

while the 4D massless propagator is

$$G_4(x, x')_{\mu\nu\rho\sigma} = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip \cdot (x-x')}}{p^2} \left( \frac{1}{2} g_{\mu\rho} g_{\nu\sigma} + \frac{1}{2} g_{\mu\sigma} g_{\nu\rho} - \frac{1}{2} g_{\mu\nu} g_{\rho\sigma} \right) . \quad (2)$$

The difference in the tensor structure of these two propagators is caused by the presence of an extra polarization state, a 4D scalar field, contained in the 5D graviton propagator. Thus we see that the situation in theories with localization and quasi-localization of gravity is not very different: In the case of localized gravity the scalar field must be included in the effective 4D theory, while in the case of quasi-localized gravity the scalar is eaten by the massive graviton modes and appears as an additional graviton polarization. One has to explain in both cases why the extra polarization state does not eventually contribute to the propagator on the brane. Intuitively this is the case for the following reason: in brane models the stress tensor for a source on a brane has a vanishing 55 component; thus we might naïvely expect that one degree of freedom, namely the scalar field, decouples from the sources on the brane and therefore the ordinary massless 4D graviton propagator should be reproduced. It has been shown in two beautiful papers by Garriga and Tanaka [16], and by Giddings, Katz and Randall [17] that this is in fact the case. They have shown that once a source term on the brane is introduced the brane itself is bent in a frame in which gravitational fluctuations are small. The effect of the bending is such that it exactly compensates for the effect of the extra 4D scalar field contained in the 5D graviton propagator, and thus the ordinary 4D massless graviton propagator is reproduced in the RS model. This hints that a similar situation may occur for theories with quasi-localized gravity, since the massive propagator of the KK modes making up the resonance exactly the same tensor structure as the massless graviton plus the scalar. We will show that the effect of the brane bending will again cancel the effects of the extra polarization in the massive propagator, up to corrections depending on the width of the resonance, which can be made arbitrarily small by adjusting the parameters of the theory. Thus there is no discontinuity in these models as  $\Delta m \rightarrow 0$  and the results of ordinary 4D Einstein theory are reproduced at intermediate scales up to small corrections.

We now turn to the problem of constructing the propagators in a general brane world described by the metric

$$ds^2 = dy^2 + e^{-A(y)} \eta_{\mu\nu} dx^\mu dx^\nu . \quad (3)$$

We will take  $A(y)$  to approach the Randall-Sundrum form for  $|y| \ll y_0$ :

$$A(y) \rightarrow 2k|y| , \quad (4)$$

while for  $|y| \gg y_0$  the metric becomes flat:

$$A(y) \rightarrow \text{constant} . \quad (5)$$

For instance, this is achieved in the GRS model [23] by simply patching anti-de Sitter space to flat space at some point  $y = y_0$ :

$$A(y) = \begin{cases} 2k|y| & |y| \leq y_0 \\ 2ky_0 & |y| \geq y_0 \end{cases} . \quad (6)$$

However, more generally one can imagine backgrounds that smoothly interpolate between  $AdS_5$  and flat space [25]. Since the metric approaches the RS metric for  $|y| \ll y_0$ , there is a brane located at  $y = 0$  on which the matter fields will live.

We now study gravitational fluctuations in the background (3). It is possible to choose a gauge for the gravitational fluctuations so that they have the form

$$ds^2 = dy^2 + e^{-A(y)}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu , \quad (7)$$

where, in addition, we impose the transverse-traceless conditions

$$\partial^\nu h_{\mu\nu} = h^\mu_\mu \equiv h = 0 . \quad (8)$$

In this gauge, the transverse-traceless fluctuations satisfy the simple equation

$$(e^A \square^{(4)} + \partial_y^2 - 2A' \partial_y) h_{\mu\nu} = 0 , \quad (9)$$

which is nothing but the scalar wave equation for each of the components  $h_{\mu\nu}$  in the background (3). At the brane, we have the usual matching condition:

$$\partial_y h_{\mu\nu} \big|_{y=0} = 0 . \quad (10)$$

In the GRS model, there is an additional matching condition at the point  $y_0$ , where  $AdS_5$  is patched onto flat space:

$$\partial_y h_{\mu\nu} \big|_{y=y_0+} = \partial_y h_{\mu\nu} \big|_{y=y_0-} . \quad (11)$$

Until now, the only matter sources in the theory were those that were needed to produce the non-trivial background (3).<sup>3</sup> Now suppose we have additional sources corresponding to matter on the brane. The question is how this modifies the equations for the fluctuations. It turns out the answer, as described in the papers by Garriga and Tanaka [16], and by Giddings, Katz and Randall [17], is rather subtle but crucial for our analysis. We closely follow the former paper in what follows.

The point is that in the original coordinate system  $(x^\mu, y)$ , the presence of the source on the brane actually bends the brane so that is no longer situated at  $y = 0$ . In order to investigate

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<sup>3</sup>The details of these background sources is not important for our present discussion. We only need assume that the sources vary linearly with a variation of the metric (see [8] for a discussion of this).

this effect, it is useful to introduce a new coordinate system  $(\bar{x}^\mu, \bar{y})$  defined so that the brane is at  $\bar{y} = 0$ . We require that the metric in the new coordinate system also has the form (7), and this means that the two coordinate systems must be related via

$$\bar{y} = y + \hat{\xi}^y(x) , \quad \bar{x}^\mu = x^\mu + \hat{\xi}^\mu(x) - \partial^\mu \hat{\xi}^y(x) \int^y dy' e^{A(y')} . \quad (12)$$

Notice that the functions  $\hat{\xi}^y(x)$  and  $\hat{\xi}^\mu(x)$  do not depend on  $y$ . The relation between the metric fluctuations in the two coordinates systems is then

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \partial_\mu \hat{\xi}_\nu + \partial_\nu \hat{\xi}_\mu - 2\partial_\mu \partial_\nu \hat{\xi}^y \int^y dy' e^{A(y')} - \eta_{\mu\nu} \hat{\xi}^y \partial_y A . \quad (13)$$

At the moment, we have not specified the function  $\hat{\xi}^y(x)$  and  $\hat{\xi}^\mu(x)$ ; however, they will be determined self-consistently, as we shall see below.

The additional sources will modify the matching condition at the brane (10). In the new coordinate system, where the brane is at  $\bar{y} = 0$ , the Israel equations at the brane give,

$$\partial_{\bar{y}} \bar{h}_{\mu\nu} \big|_{\bar{y}=0+} = -\kappa^2 (S_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} S) , \quad (14)$$

where  $S_{\mu\nu}$  is given in terms of the matter stress tensor via

$$T_{\mu\nu}^{\text{brane}} = S_{\mu\nu}(x) \delta(\bar{y}) , \quad T_{\bar{y}\bar{y}}^{\text{brane}} = T_{\mu\bar{y}}^{\text{brane}} = 0 . \quad (15)$$

Using the relation between the metric fluctuations (13) we can now write down the boundary condition (14) in the original coordinates:<sup>4</sup>

$$\partial_y h_{\mu\nu} \big|_{y=0+} = -\kappa^2 (S_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} S) - 2\partial_\mu \partial_\nu \hat{\xi}^y . \quad (16)$$

In the GRS model there is also the matching condition at  $y_0$  (11). It is easy to show that this condition is simply

$$\partial_{\bar{y}} \bar{h}_{\mu\nu} \big|_{\bar{y}=\bar{y}_0+} = \partial_{\bar{y}} \bar{h}_{\mu\nu} \big|_{\bar{y}=\bar{y}_0-} , \quad (17)$$

and so in the original coordinates (11) remains unchanged.

The position of the brane in the original coordinate system is  $y = -\hat{\xi}^y(x)$  and so the condition (16) includes, via  $\hat{\xi}^y(x)$ , an effect from the bending of the brane. We now can now combine the equation of motion (9) and the boundary condition (16) to give

$$(e^A \square^{(4)} + \partial_y^2 - 2A' \partial_y) h_{\mu\nu} = -2\kappa^2 \Sigma_{\mu\nu} \delta(y) , \quad (18)$$

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<sup>4</sup>Notice that to leading order in the source we can specify the following condition at  $y = 0$ .

where we have defined the effective source

$$\Sigma_{\mu\nu} = S_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}S + \frac{2}{\kappa^2}\partial_\mu\partial_\nu\hat{\xi}^y . \quad (19)$$

In order to determine  $\hat{\xi}^y$ , we now impose the fact that  $h_{\mu\nu}$  is traceless:  $h = 0$ . This implies that the right-hand side of (18) itself must be traceless and so

$$\square^{(4)}\hat{\xi}^y = \frac{\kappa^2}{6}S , \quad (20)$$

with solution

$$\hat{\xi}^y(x) = \frac{\kappa^2}{6} \int d^4x' \Delta_4(x, x') S(x') , \quad (21)$$

where  $\Delta_4(x, x')$  is the massless scalar Green's function for 4D Minkowski space. If we now define the five-dimensional Green's function

$$\left( e^A \square^{(4)} + \partial_y^2 - 2A' \partial_y \right) \Delta_5(x, y; x', y') = \delta^{(4)}(x - x') \delta(y - y') , \quad (22)$$

then the fluctuation due to the source on the brane can be written

$$h_{\mu\nu}(x, y) = -2\kappa^2 \int d^4x' \Delta_5(x, y; x', 0) \Sigma_{\mu\nu}(x') . \quad (23)$$

The final thing that we need to do is to transform the fluctuation back into the coordinate system  $(\bar{x}^\mu, \bar{y})$  in which the brane is straight. In order to have a simple final expression for the fluctuation on the brane, we can use the additional freedom in the transformation (12), present in  $\hat{\xi}^\mu(x)$ , to set,

$$\hat{\xi}_\mu(x) = \partial_\mu \left( \frac{1}{2k} \hat{\xi}^y(x) - 2 \int d^4x' \Delta_5(x, 0; x', 0) \hat{\xi}^y(x') \right) . \quad (24)$$

Then from (13) and (23) we obtain our final result for the fluctuation evaluated on the brane:

$$\bar{h}_{\mu\nu}(x, 0) = -2\kappa^2 \int d^4x' \left\{ \Delta_5(x, 0; x', 0) (S_{\mu\nu}(x') - \frac{1}{3}\eta_{\mu\nu}S(x')) - \frac{k}{6} \Delta_4(x, x') \eta_{\mu\nu} S(x') \right\} , \quad (25)$$

where  $k = A'(0)/2$ . Note that in a flat background the last term in (25) would be absent, and the theory is simply that of five-dimensional gravity, as expected.<sup>5</sup> The first term is exactly what one would have naïvely expected, while the second term arises from the effect of the bending of the brane [16, 17].

Notice that the result (25) is written in terms of the scalar Green's function  $\Delta_5(x, y; x', y')$  for the background (3) which allows us to make contact with the auxiliary quantum mechanical system of [1]. First of all, we relate the coordinate  $z$  to  $y$  via

$$\frac{dz}{dy} = e^{A(y)/2} . \quad (26)$$

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<sup>5</sup>We thank John Terning for raising this issue.

The wavefunctions  $\psi_m(z)$ , which satisfy the Schrödinger equation [25]

$$-\frac{d^2\psi_m}{dz^2} + \left(\frac{9}{16}A'^2 - \frac{3}{4}A''\right)\psi_m = m^2\psi_m , \quad (27)$$

where  $' \equiv d/dz$ , and then give the Green's function that we need in (25). It is expressed as a sum over the bound-states and an integral over the continuum of KK modes:

$$\Delta_5(x, 0; x', 0) = \int \frac{d^4p}{(2\pi)^4} e^{ip \cdot (x-x')} \left\{ \sum_m \frac{\psi_m(0)^2}{p^2 + m^2} + \int dm \frac{\psi_m(0)^2}{p^2 + m^2} \right\} . \quad (28)$$

The Schrödinger equation (27) always admits a zero energy state

$$\hat{\psi}_0(z) = N_0 e^{-3A(z)/4} , \quad (29)$$

which potentially describes the 4D graviton. In the RS model

$$\hat{\psi}_0(z) = \frac{k^{1/2}}{(k|z| + 1)^{3/2}} , \quad (30)$$

is normalizable and so

$$\Delta_5(x, 0; x', 0) = k\Delta_4(x, x') + \dots , \quad (31)$$

where the ellipsis represents the contribution from the KK modes. The contribution of the massless graviton fluctuation in (25) is then,

$$\bar{h}_{\mu\nu}(x, 0) = -2\kappa^2 k \int d^4x' \Delta_4(x, x') (S_{\mu\nu}(x') - \frac{1}{3}\eta_{\mu\nu}S(x')) + \dots . \quad (32)$$

However, this is *not* the usual propagator of a massless 4D graviton. Fortunately, (32) does not include the effect of the brane bending term in (25), which provides an additional contribution that precisely has the effect of changing  $\frac{1}{3} \rightarrow \frac{1}{2}$  in (32) and so yields the usual massless 4D graviton propagator. So the brane bending effect is crucial even in the original RS model [16, 17].

In a quasi-localized gravity model, the transverse space is asymptotically flat, *i.e.*  $A(y) \rightarrow \text{constant}$  for  $|y| \gg y_0$ . In this case, the state  $\hat{\psi}_0(z)$  is not normalizable and there are only contributions to the Green's function from continuum modes. However, in [25] we argued that if the scale  $y_0$  is sufficiently large then  $\hat{\psi}_0(z)$  appears as a sharp resonance at the bottom of the continuum at  $m = 0$ . In this case, we have approximately, for small  $m$ ,

$$\psi_m(0)^2 = \frac{\mathcal{A}}{m^2 + \Delta m^2} + \dots . \quad (33)$$

The height of the resonance is given by

$$\frac{\mathcal{A}}{\Delta m^2} = \hat{\psi}_0(0)^2 = e^{-3(A(0)-A(\infty))/2} , \quad (34)$$

while the width, to leading order, can be determined by the fact that as  $y_0 \rightarrow \infty$ , the model should reduce to the RS model where  $\hat{\psi}_0(z)$  is normalizable. So as  $\Delta m \rightarrow 0$ , (33) should approximate  $k\delta(m)$  giving

$$\frac{\mathcal{A}}{\Delta m} = \frac{2k}{\pi} . \quad (35)$$

In the GRS model (34) and (35) give [23]

$$\Delta m = \frac{2k}{\pi} e^{-3ky_0} . \quad (36)$$

For small, but non-zero  $\Delta m$ , *i.e.* large  $y_0 \gg k^{-1}$ , the resonance can mimic the effect of the bound-state in the RS model. For  $|x - x'| \ll \Delta m^{-1}$ , we can approximate the effect of the resonance by a delta function and hence naïvely we would expect a contribution to the graviton propagator as in (32). However, just as in the RS model itself we have to include the effect of the brane bending. This provides an additional contribution which precisely has the effect of changing  $\frac{1}{3} \rightarrow \frac{1}{2}$  in (32) and yields the usual massless 4D graviton propagator [16,17]. So the effect of brane bending in the case of quasi-localized gravity on the brane is exactly the same as for the RS model. We recover the normal 4D graviton propagator in the regime where we can ignore the contribution from the rest of the continuum,  $|x - x'| \gg k^{-1}$ , and when we can approximate the resonance by a delta function,  $|x - x'| \ll \Delta m^{-1}$ . The effect of the finite width will give a calculable correction to the graviton propagator that can be determined by integrating over the shape of the resonance, giving

$$\begin{aligned} \bar{h}_{\mu\nu}(x, 0) = & -2\kappa^2 k \int d^4 x' \left\{ \Delta_4(x, x') (S_{\mu\nu}(x') - \tfrac{1}{2} \eta_{\mu\nu} S(x')) \right. \\ & \left. + \Delta m \tilde{\Delta}_4(x, x') (S_{\mu\nu}(x') - \tfrac{1}{3} \eta_{\mu\nu} S(x')) \right\} , \end{aligned} \quad (37)$$

where

$$\tilde{\Delta}_4(x, x') = - \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ip \cdot (x - x')}}{p^2(p + \Delta m)} . \quad (38)$$

We have studied theories in which gravity is quasi-localized to a brane. In these theories there is a massless graviton resonance which eventually decays into the bulk, with the result of altering the very long distance behavior of gravity. It has been suggested that there are phenomenological difficulties with such models as a result of the non-decoupling of massive graviton polarizations in the massless limit. In this letter we have shown that in fact such difficulties are absent. The reason is that the bending of the brane exactly compensates for the effects of the extra polarization in the massive graviton propagator (just as it compensates for the effect of the massless scalar in the case of the RS theory). Thus the graviton propagator at intermediate distances will be equal to the massless propagator of the Einstein theory (up to corrections that can be made arbitrarily small by making the width of the resonance small). Therefore, all classic predictions of general relativity



including the bending of light around the Sun and the precession rate of the orbit of Mercury are correctly reproduced in these theories.

Theories with quasi-localized gravity open exciting possibilities for phenomenology. Because gravity is modified at both very short and very long distances in such models, there are phenomenological consequences for both particle physics at high energies and cosmology at large distances. These consequences merit further investigation.

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## References

- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [[hep-th/9906064](#)].
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [[hep-ph/9905221](#)].
- [3] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000) [[hep-th/9907209](#)].
- [4] C. Csáki and Y. Shirman, Phys. Rev. **D61** 024008 (2000) [[hep-th/9908186](#)]; A. E. Nelson, [hep-th/9909001](#).
- [5] S. M. Carroll, S. Hellerman and M. Trodden, [hep-th/9911083](#). S. Nam, [hep-th/9911104](#).
- [6] K. Behrndt and M. Cvetič, [hep-th/9909058](#); O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, [hep-th/9909134](#); A. Chamblin and G. W. Gibbons, [hep-th/9909130](#).
- [7] M. Gremm, [hep-th/9912060](#).
- [8] C. Csáki, J. Erlich, T. J. Hollowood and Y. Shirman, [hep-th/0001033](#).
- [9] M. Gremm, [hep-th/0002040](#).
- [10] R. Kallosh and A. Linde, JHEP **0002**, 005 (2000) [[hep-th/0001071](#)].
- [11] H. Verlinde, [hep-th/9906182](#).
- [12] K. Skenderis and P. K. Townsend, Phys. Lett. **B468**, 46 (1999) [[hep-th/9909070](#)].
- [13] A. Brandhuber and K. Sfetsos, JHEP **9910**, 013 (1999) [[hep-th/9908116](#)].
- [14] S.S. Gubser, [hep-th/9912001](#).

- [15] A. Chamblin, S.W. Hawking and H.S. Reall, [hep-th/9909205](#); R. Emparan, G. Horowitz and R. Myers, [hep-th/9911043](#); [hep-th/9912135](#); A. Chamblin, C. Csáki, J. Erlich and T. J. Hollowood, [hep-th/0002076](#).
- [16] J. Garriga and T. Tanaka, [hep-th/9911055](#).
- [17] S. B. Giddings, E. Katz and L. Randall, [hep-th/0002091](#).
- [18] T. Shiromizu, K. Maeda and M. Sasaki, [gr-qc/9910076](#); [hep-th/9912233](#); J. Garriga and M. Sasaki, [hep-th/9912118](#); S. Mukohyama, T. Shiromizu and K. Maeda, [hep-th/9912287](#).
- [19] J. Cline, C. Grojean and G. Servant, [hep-ph/9909496](#); C. Grojean, J. Cline and G. Servant, [hep-th/9910081](#); C. Grojean, [hep-th/0002130](#).
- [20] W. Muck, K. S. Viswanathan and I. V. Volovich, [hep-th/0002132](#); Y. S. Myung, G. Kang and H. W. Lee, [hep-th/0001107](#); M. G. Ivanov and I. V. Volovich, [hep-th/9912242](#); S. Myung and G. Kang, [hep-th/0001003](#).
- [21] V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **125B**, 139 (1983); M. Visser, Phys. Lett. **159B**, 22 (1985); E.J. Squires, Phys. Lett. **167B**, 286 (1986); G.W. Gibbons and D.L. Wiltshire, Nucl. Phys. **B287**, 717 (1987); M. Gogberashvili, [hep-ph/9812296](#); [hep-ph/9812365](#); [hep-ph/9904383](#); [hep-ph/9908347](#).
- [22] M. Cvetič, S. Griffies and S. Rey, Nucl. Phys. **381**, 301 (1992) [[hep-th/9201007](#)]. For a review see: M. Cvetič and H.H. Soleng, Phys. Rept. **282**, 159 (1997) [[hep-th/9604090](#)].
- [23] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, [hep-th/0002072](#).
- [24] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross and J. Santiago, [hep-ph/9912552](#).
- [25] C. Csáki, J. Erlich and T. J. Hollowood, [hep-th/0002161](#).
- [26] G. Dvali, G. Gabadadze and M. Porrati, [hep-th/0002190](#).
- [27] E. Witten, [hep-ph/0002297](#).
- [28] H. van Dam and M. Veltman, Nucl. Phys. **B22**, 397 (1970); V.I. Zakharov, JETP Lett. **12**, 312 (1970).